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An oscillation theorem for a class of second-order forced neutral delay differential equations with mixed nonlinearities[☆]

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ABSTRACT

A class of second-order forced neutral delay differential equation with mixed nonlinearities

$$(r(t)|x'(t)+px'(t-\sigma)|^{\alpha-1}(x'(t)+px'(t-\sigma)))'+q_0(t)|x(\tau_0(t))|^{\alpha-1}x(\tau_0(t))$$
$$+\sum_{i=1}^{n}q_i(t)|x(\tau_i(t))|^{\beta_i-1}x(\tau_i(t))=e(t)\mathrm{sgn}(x(t))$$

is investigated in this paper. Using a new method, we obtain some new sufficient conditions for the oscillation of the above equation, and some known oscillation criteria have been extended.

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1. Introduction

Consider the following second-order forced neutral delay differential equations with mixed nonlinearities:

$$(r(t)|x'(t)+px'(t-\sigma)|^{\alpha-1}(x'(t)+px'(t-\sigma)))'+q_0(t)|x(\tau_0(t))|^{\alpha-1}x(\tau_0(t))$$
$$+\sum_{i=1}^{n}q_i(t)|x(\tau_i(t))|^{\beta_i-1}x(\tau_i(t))=e(t)\mathrm{sgn}(x(t)),\quad t\geq t_0, \tag{1}$$

where $\alpha\leq\beta_i$ $(i=1,2,\ldots,n)$ are positive constants, $r(t)\in C^1([t_0,\infty);R^+)$, $r'(t)\geq 0$, $q_i(t)\geq 0$ $(i=0,1,2,\ldots,n)$, $e(t)\leq 0$, $p\geq 0$, and $\sigma\geq 0$. Throughout this paper, we suppose that there exists $\tau(t)\in C^1([t_0,\infty);R^+)$ such that $\tau(t)\leq\tau_i(t)$ $(i=0,1,2,\ldots,n)$, $\tau(t)\leq t-\sigma$, $\lim_{t\to\infty}\tau(t)=\infty$, and $\tau'(t)>0$ for $t\in[t_0,\infty)$.

Many authors have investigated the oscillation of second-order differential equations [1–9]; Li and Cheng [10] have obtained oscillation criteria for a special case of (1):

$$(r(t)|x'(t)|^{\alpha-1}x'(t))'+p(t)|x(t)|^{\alpha-1}x(t)=e(t). \tag{2}$$

More recently, Zheng and Wang [11] have investigated some similar properties for the more common form of (2):

$$(r(t)|x'(t)|^{\alpha-1}x'(t))'+p(t)|x(t)|^{\alpha-1}x(t)+\sum_{i=1}^{n}q_i(t)|x(t)|^{\beta_i-1}x(t)=e(t). \tag{3}$$

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However, to the best of the authors' knowledge, there are few works on second-order forced neutral delay differential equations of the above form.

In this paper, we investigate the oscillation of (1); we restrict our attention to those solutions $x(t)$ of (1) which exist on some half line $[t_x, \infty]$ and satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for any $T \geq t_x$.

A solution $x(t)$ of (1) is oscillatory if and only if it has arbitrarily large zeros; otherwise, it is non-oscillatory. If a solution is non-oscillatory, then it is eventually positive or eventually negative. (1) is oscillatory if and only if every solution of (1) is oscillatory.

The paper is arranged as follows. In Section 2, we introduce a lemma, which will be used in Section 3. We obtain our main results in Section 3. Finally, an example is given to explain our results.

2. Our lemma

To obtain our results, we introduce a lemma as follows.

Lemma 2.1 (*See [12]*). *Let $f, g : [t_0, \infty) \to R$, such that*

$$f(t) = g(t) + pg(t - c), \quad t \geq t_0 + \max\{0, c\},$$

where $p, c \in R$ and $p \neq 1$.

Assume that $\lim_{t\to\infty} f(t) \equiv l \in R$ exists. Then the following statements hold.

1. *If $\liminf_{t\to\infty} g(t) \equiv a \in R$, then $l = (1 + p)a$.*
2. *If $\limsup_{t\to\infty} g(t) \equiv b \in R$, then $l = (1 + p)b$.*

3. Main results

Theorem 3.1. *Assume that $0 \leq p < \infty, p \neq 1$. If*

$$\lim_{t\to\infty} R(t) = \infty, \tag{4}$$

and there exists an $\varepsilon (0 < \varepsilon < 1)$ and a positive continuous function $\rho(t)$ such that

$$\int_{t_0}^{\infty} \left[\frac{(1-\varepsilon)^\alpha}{(1 + p(1+\varepsilon))^\alpha} \rho(t) Q(t) - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1} r(\tau(t))}{\rho^\alpha(t)(\tau'(t))^\alpha} \right] \mathrm{d}t = \infty, \tag{5}$$

where

$$R(t) = \int_{t_0}^{t} \frac{1}{r^{1/\alpha}(s)} \mathrm{d}s, \tag{6}$$

$$Q(t) = q_0(t) + \sum_{i=1}^{n} Q_i(t) = q_0(t) + \sum_{i=1}^{n} \alpha^{-\alpha/\beta_i} \beta_i [n(\beta_i - \alpha)]^{(\alpha-\beta_i)/\beta_i} (q_i(t))^{\alpha/\beta_i} |e(t)|^{(\beta_i-\alpha)/\beta_i}, \tag{7}$$

$$\rho'_+(t) = \max\{0, \rho'(t)\}, \tag{8}$$

then (1) is oscillatory.

Proof. Assume that (1) has a non-oscillatory solution $x(t)$. Without loss of generality, suppose that it is an eventually positive solution (if it is an eventually negative solution, the proof is similar). Define

$$y(t) = x(t) + px(t - \sigma). \tag{9}$$

Then (1) can be rewritten as the following:

$$(r(t)|y'(t)|^{\alpha-1}y'(t))' + q_0(t)|x(\tau_0(t))|^{\alpha-1}x(\tau_0(t)) + \sum_{i=1}^{n} q_i(t)|x(\tau_i(t))|^{\beta_i-1}x(\tau_i(t)) = e(t) \quad \text{for } t \geq t_0. \tag{10}$$

It is obvious that there exists a $t_1 \geq t_0$ such that $x(t) > 0, y(t) > 0$. From (10), it follows that $r(t)|y'(t)|^{\alpha-1}y'(t)$ is eventually decreasing, which means that $y'(t)$ is eventually negative or eventually positive.

Now, we claim that $y'(t) > 0$. Otherwise, $y'(t) \leq 0$. Since $x(t)$ is eventually positive, using (10), we can easily show that $[r(t)(-y'(t))^\alpha]' = [r(t)| - y'(t)|^{\alpha-1}(-y'(t))]' \geq 0$. Because $r(t)(-y'(t))^\alpha$ is eventually positive, there exist $M > 0$ and $t_2 \geq t_1$ such that

$$r(t)(-y'(t))^\alpha \geq M, \quad t \geq t_2;$$

that is,

$$-y'(t) \geq M^{1/\alpha} \frac{1}{r^{1/\alpha}(t)}, \quad t \geq t_2.$$

Integrating the above inequality, taking the limit as $t \to \infty$, and using (4) and (6), we obtain that $\lim_{t\to\infty} y(t) \leq -\infty$, which contradicts the fact that $y(t)$ is eventually positive.

Hence, we have

$$x(t) > 0, \qquad y(t) > 0, \qquad y'(t) > 0, \qquad [r(t)(y'(t))^{\alpha}]' \leq 0, \quad t > t_3,$$

and it follows that

$$r(t)(y'(t))^{\alpha} \leq r(\tau(t))(y'(\tau(t)))^{\alpha}, \quad t \geq t_3,$$

which implies that

$$\frac{y'(\tau(t))}{y'(t)} \geq \left(\frac{r(t)}{r(\tau(t))}\right)^{1/\alpha}, \quad t \geq t_3. \tag{11}$$

Define

$$u(t) = \rho(t) \frac{r(t)(y'(t))^{\alpha}}{(y(\tau(t)))^{\alpha}}, \quad t \geq t_3. \tag{12}$$

Then $u(t) > 0$.

Because $y'(t) > 0$ and $[r(t)(y'(t))^{\alpha}]' \leq 0$, this means that $r(t)(y'(t))^{\alpha}$ is a monotone decrease function. Because $r(t)$ is increasing, $y'(t)$ is decreasing. Therefore $\lim_{t\to\infty} y'(t) = b \geq 0$.

It is easy to show that $x'(t)$ is bounded. By Lemma 2.1, we obtain that $\liminf_{t\to\infty} x'(t) = \limsup_{t\to\infty} x'(t) = \lim_{t\to\infty} x'(t) = a$, and $a = b/(1+p) \geq 0$. We consider the following two cases.

Case 1. $\lim_{t\to\infty} x'(t) = a > 0$. Then there exists a $t_3 > t_2$ such that $x'(t) > 0$ for $t > t_3$. From (10) and (12), and noticing that $x'(t) > 0$, we have

$$\begin{aligned} u'(t) &= \frac{\rho'(t)}{\rho(t)} u(t) - \alpha\rho(t) \frac{r(t)(y'(t))^{\alpha}}{(y(\tau(t)))^{\alpha+1}} y'(\tau(t))\tau'(t) - \rho(t) \frac{q_0(t)x^{\alpha}(\tau_0(t)) + \sum_{i=1}^{n} q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{(x(\tau(t)) + px(\tau(t)-\sigma))^{\alpha}} \\ &\leq \frac{\rho'_+(t)}{\rho(t)} u(t) - \frac{\alpha\tau'(t)}{(\rho(t)r(\tau(t)))^{1/\alpha}} u^{(\alpha+1)/\alpha}(t) - \frac{1}{(1+p)^{\alpha}} \rho(t) \left\{ q_0(t) + \sum_{i=1}^{n} q_i(t)x^{\beta_i-\alpha}(\tau(t)) - \frac{e(t)}{x^{\alpha}(\tau(t))} \right\}, \\ &\quad t > t_3. \end{aligned} \tag{13}$$

Define

$$F(s) = \frac{\rho'_+(t)}{\rho(t)} s - \frac{\alpha\tau'(t)}{(\rho(t)r(\tau(t)))^{1/\alpha}} s^{(\alpha+1)/\alpha}, \quad s > 0.$$

Now, we find the maximum point of $F(s)$; that is, there exists a point s_0 such that

$$F'(s_0) = 0, \qquad F''(s_0) < 0.$$

It is easy to show that

$$s_0 = \frac{1}{(\alpha+1)^{\alpha}} \frac{(\rho'_+(t))^{\alpha}}{\rho^{\alpha-1}(t)} \frac{r(\tau(t))}{(\tau'(t))^{\alpha}}.$$

Hence, $F(s)$ have it maximum on s_0. So, we have

$$F(s) \leq F(s_0) = \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1}}{\rho^{\alpha}(t)} \frac{r(\tau(t))}{(\tau'(t))^{\alpha}}. \tag{14}$$

Set

$$F_i(x) = q_i(t)x^{\beta_i-\alpha} - \frac{e(t)}{nx^{\alpha}}, \quad 1 \leq i \leq n.$$

Now, we find the minimum point of $F_i(x)$; that is, there exists a point x_i^* such that

$$F_i'(x_i^*) = 0, \qquad F_i''(x_i^*) > 0.$$

It is obvious that

$$x_i^* = \left[\frac{-\alpha e(t)}{n(\beta_i - \alpha)q_i(t)}\right]^{1/\beta_i}.$$

Hence $F_i(x)$ has its minimum on x_i^*, and

$$F_i(x) \geq F_i(x_i^*) = Q_i(t). \tag{15}$$

Combining (13)–(15), it follows that

$$u'(t) \leq \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(t))^{\alpha+1}}{\rho^\alpha(t)} \frac{r(\tau(t))}{(\tau'(t))^\alpha} - \frac{1}{(1+p)^\alpha}\rho(t)Q(t), \quad t > t_3, \tag{16}$$

where $Q(t)$ is defined in (7).

Integrating both sides of the above inequality from t_3 to t, we have

$$\begin{aligned} 0 < u(t) &\leq u(t_3) - \int_{t_3}^{t} \left(\frac{1}{(1+p)^\alpha}\rho(s)Q(s) - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(s))^{\alpha+1}}{\rho^\alpha(s)} \frac{r(\tau(s))}{(\tau'(s))^\alpha}\right) \mathrm{d}s \\ &< u(t_3) - \int_{t_3}^{t} \left(\frac{(1-\varepsilon)^\alpha}{(1+p(1+\varepsilon))^\alpha}\rho(s)Q(s) - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(s))^{\alpha+1}}{\rho^\alpha(s)} \frac{r(\tau(s))}{(\tau'(s))^\alpha}\right) \mathrm{d}s. \end{aligned} \tag{17}$$

Taking $t \to \infty$ in (17), and using (5), it follows that

$$0 < \lim_{t\to\infty} u(t) \leq -\infty.$$

This is a contradiction.

Case 2. $\lim_{t\to\infty} x'(t) = a = 0$. Because $y(t) > 0$ and $y'(t) > 0$, $\lim_{t\to\infty} y(t) = \bar{b} > 0$, and $\lim_{t\to\infty} x(t) = \bar{a} = \frac{\bar{b}}{1+p} > 0$, it follows that $\lim_{t\to\infty} \frac{x(\tau_i(t))}{x(\tau(t))} = 1$, $\lim_{t\to\infty} \frac{x(\tau(t)-\sigma)}{x(\tau(t))} = 1$, which means that, for any positive number $\varepsilon' < \varepsilon$, there exists a $T > t_3$ such that $1 - \varepsilon' < \frac{x(\tau_i(t))}{x(\tau(t))}, \frac{x(\tau(t)-\sigma)}{x(\tau(t))} < 1 + \varepsilon'$, $i = 0, 1, \ldots, n$, $t > T$. Thus, (13) can be rewritten as

$$\begin{aligned} u'(t) &= \frac{\rho'(t)}{\rho(t)}u(t) - \alpha\rho(t)\frac{r(t)(y'(t))^\alpha}{(y(\tau(t)))^{\alpha+1}}y'(\tau(t))\tau'(t) \\ &\quad - \frac{1}{\left(1 + p\frac{x(\tau(t)-\sigma)}{x(\tau(t))}\right)^\alpha}\rho(t)\frac{q_0(t)x^\alpha(\tau_0(t)) + \sum_{i=1}^{n} q_i(t)x^{\beta_i}(\tau_i(t)) - e(t)}{(x(\tau(t)))^\alpha} \\ &\leq \frac{\rho'_+(t)}{\rho(t)}u(t) - \frac{\alpha\tau'(t)}{(\rho(t)r(\tau(t)))^{1/\alpha}}u^{(\alpha+1)/\alpha}(t) \\ &\quad - \frac{(1-\varepsilon')^\alpha}{(1+p(1+\varepsilon'))^\alpha}\rho(t)\left\{q_0(t) + \sum_{i=1}^{n} q_i(t)x^{\beta_i-\alpha}(\tau(t)) - \frac{e(t)}{x^\alpha(\tau(t))}\right\}, \quad t > T. \end{aligned}$$

Using a similar method as in Case 1, it is easy to show that

$$\begin{aligned} 0 < u(t) &\leq u(T) - \int_{T}^{t} \left(\frac{(1-\varepsilon')^\alpha}{(1+p(1+\varepsilon'))^\alpha}\rho(s)Q(s) - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(s))^{\alpha+1}}{\rho^\alpha(s)} \frac{r(\tau(s))}{(\tau'(s))^\alpha}\right) \mathrm{d}s \\ &\leq u(T) - \int_{T}^{t} \left(\frac{(1-\varepsilon)^\alpha}{(1+p(1+\varepsilon))^\alpha}\rho(s)Q(s) - \frac{1}{(\alpha+1)^{\alpha+1}} \frac{(\rho'_+(s))^{\alpha+1}}{\rho^\alpha(s)} \frac{r(\tau(s))}{(\tau'(s))^\alpha}\right) \mathrm{d}s. \end{aligned}$$

Taking $t \to \infty$ in the above, and using (5), it follows that

$$0 < \lim_{t\to\infty} u(t) \leq -\infty.$$

This is also a contradiction. The proof is complete. □

Remark 3.1. If we choose $\rho(t) = R^\alpha(\tau(t))$, then (5) reduces to

$$\int_{t_0}^{\infty} \left[\frac{(1-\varepsilon)^\alpha}{(1+p(1+\varepsilon))^\alpha}R^\alpha(\tau(t))Q(t) - \left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1} \frac{\tau'(t)}{R(\tau(t))}r^{1/\alpha}(\tau(t))\right] \mathrm{d}t = \infty.$$

4. An example

Example 4.1. Consider the following equation:

$$\left((2+\cos t)\left|x'(t)+\frac{1}{2}x'(t-\pi)\right|^{-\frac{2}{3}}\left(x'(t)+\frac{1}{2}x'(t-\pi)\right)\right)'+3x(t-\pi)=-\cos^2 t\,\mathrm{sgn}(x(t)),\quad t\geq t_0. \tag{18}$$

It is obvious that $\alpha=\frac{1}{3},\ \beta=1,\ p=\frac{1}{2},\ q(t)=3,\ \tau(t)=t-\pi,\ r(t)=2+\cos t$, and $e(t)=\cos^2 t$. It is easy to check that

$$\lim_{t\to\infty}R(t)=\int_{t_0}^{\infty}\frac{1}{r^{1/\alpha}(t)}\mathrm{d}t=\int_{t_0}^{\infty}\frac{1}{(2+\cos t)^3}\mathrm{d}t=\infty.$$

Take $\rho(t)=t$; then $\rho'(t)=1$. Since

$$Q(t)=\left(\frac{1}{3}\right)^{-\frac{1}{3}}\left(\frac{2}{3}\right)^{-\frac{2}{3}}3^{\frac{1}{3}}(\cos^2 t)^{\frac{2}{3}}=3^{\frac{4}{3}}4^{-\frac{1}{3}}\cos^{\frac{4}{3}}t,\qquad r(\tau(t))=2+\cos(t-\pi)=2-\cos t,$$

for any $0<\varepsilon<1$, we have

$$\begin{aligned}
&\int_{t_0}^{\infty}\left[\frac{(1-\varepsilon)^{\alpha}}{(1+p(1+\varepsilon))^{\alpha}}\rho(t)Q(t)-\frac{1}{(\alpha+1)^{\alpha+1}}\frac{(\rho'_+(t))^{\alpha+1}r(\tau(t))}{\rho^{\alpha}(t)(\tau'(t))^{\alpha}}\right]\mathrm{d}t\\
&\quad=\int_{t_0}^{\infty}\left[\frac{(1-\varepsilon)^{\alpha}}{\left(1+\frac{1}{2}(1+\varepsilon)\right)^{\alpha}}3^{\frac{4}{3}}4^{-\frac{1}{3}}t\cos^{\frac{4}{3}}t-\left(\frac{3}{4}\right)^{\frac{4}{3}}\frac{2-\cos t}{t^{\frac{1}{3}}}\right]\mathrm{d}t\\
&\quad\geq\int_{t_0}^{\infty}\left[\frac{(1-\varepsilon)^{\alpha}}{\left(1+\frac{1}{2}(1+\varepsilon)\right)^{\alpha}}3^{\frac{4}{3}}4^{-\frac{1}{3}}t\cos^{\frac{4}{3}}t-\left(\frac{3}{4}\right)^{\frac{4}{3}}3t^{-\frac{1}{3}}\right]\mathrm{d}t.
\end{aligned}\tag{19}$$

Let

$$a=\frac{(1-\varepsilon)^{\alpha}}{\left(1+\frac{1}{2}(1+\varepsilon)\right)^{\alpha}}3^{\frac{4}{3}}4^{-\frac{1}{3}},\qquad b=\left(\frac{3}{4}\right)^{\frac{4}{3}}3. \tag{20}$$

Since $\int_0^{\pi}\cos^{\frac{4}{3}}t\mathrm{d}t>0$, there exists a positive integer N large enough such that

$$b\pi\left(\frac{1}{n\pi}\right)^{\frac{1}{3}}<\frac{1}{2}an\pi\int_0^{\pi}\cos^{\frac{4}{3}}t\mathrm{d}t \tag{21}$$

for $n\geq N$, and it follows that

$$\begin{aligned}
\int_{n\pi}^{(n+1)\pi}bt^{-\frac{1}{3}}\mathrm{d}t&<b\pi\left(\frac{1}{n\pi}\right)^{\frac{1}{3}}<\frac{1}{2}an\pi\int_0^{\pi}\cos^{\frac{4}{3}}t\mathrm{d}t\\
&=\frac{1}{2}an\pi\int_{n\pi}^{(n+1)\pi}\cos^{\frac{4}{3}}t\mathrm{d}t\leq\frac{1}{2}a\int_{n\pi}^{(n+1)\pi}t\cos^{\frac{4}{3}}t\mathrm{d}t
\end{aligned}\tag{22}$$

for $n\geq N$. Take $t_0=N\pi$. Substituting (22) into (19), we have from (20) that

$$\begin{aligned}
\int_{N\pi}^{\infty}\left[\frac{(1-\varepsilon)^{\alpha}}{(1+p(1+\varepsilon))^{\alpha}}\rho(t)Q(t)-\frac{1}{(\alpha+1)^{\alpha+1}}\frac{(\rho'_+(t))^{\alpha+1}r(\tau(t))}{\rho^{\alpha}(t)(\tau'(t))^{\alpha}}\right]\mathrm{d}t&\geq\Sigma_{n=N}^{\infty}\int_{n\pi}^{(n+1)\pi}\left[at\cos^{\frac{4}{3}}t-bt^{-\frac{1}{3}}\right]\mathrm{d}t\\
&\geq\frac{1}{2}\Sigma_{n=N}^{\infty}\int_{n\pi}^{(n+1)\pi}at\cos^{\frac{4}{3}}t\mathrm{d}t\\
&\geq\frac{1}{2}\left(\int_0^{\pi}\cos^{\frac{4}{3}}t\mathrm{d}t\right)\Sigma_{n=N}^{\infty}an\pi=\infty.
\end{aligned}$$

Thus, all conditions of Theorem 3.1 hold. Therefore, every solution of (18) oscillates.

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